Analysis of Longitudinal Data

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Summer Term 2016

With thanks to Anne-Laure Boulesteix for slides from previous years

Language

- The lecture and lab are held in English.
- There will be German summaries of the last lecture at the beginning of each lecture.
- As an additional service, we offer a voluntary 'Tutorium' in German.
- There will be German and English versions of the Übungsblätter/work sheets and the exam.
- You can always ask questions in German.

Dates

- Lecture (Prof. Dr. Sonja Greven): Thursday 14.15 - 15.45 (D 209) and ca. every second Tuesday, 12.15-13.45 (M 001)
- Lab session (Dipl.-Stat. Jona Cederbaum): ca. every second Tuesday, 12.15-13.45 (rooms on website)
- Tutorium (Alexander Bauer): ca. every second Tuesday, alternating with lab, 18.15-19.45 (IuK-Pool, Ludwigstr. 28, Room 207)
- Up-to-date times and rooms are on the course website `http://www.statistik.lmu.de/institut/ag/fda/ALD_2016/`.
- Sprechstunde / consultation times: by appointment

Exam

- 90 minutes long. Our proposal: 22 or 25 July 2016.
- Lectures and lab sessions are both relevant for the exam.
  The Tutorium is additional opportunity for repetition and exercise.
- The official version will be German. There will also be an English version, which can be accepted for credit instead.
- You may bring two pages with notes (front and back) in addition to a calculator and a dictionary if necessary (**not** open book).
- We offer an additional lab on 23 June going through an **old exam**.

References

- Diggle, Heagerty, Liang, and Zeger (2002). Analysis of longitudinal data. Oxford University Press.

- Fitzmaurice, Laird, Ware (2004). Applied longitudinal analysis. Wiley.

- Molenberghs and Verbeke (2005). Models for Discrete Longitudinal Data. Springer.

- Verbeke and Molenberghs (2000). Linear Mixed Models for Longitudinal Data. Springer.

Additional papers and books are referenced in the slides. A bibliography will be on the website for (voluntary) further reading.

ARSnova

- ARS stands for 'Audience Response System' and will be used for live voting and live feedback.
- Browser-based, no installation or registration required.
- Link or QR code: `https://arsnova.eu/mobile/#id/89908282`

Overview Chapter 1 - Introduction

1.1 Introduction to longitudinal data

1.2 Examples

1.3 Correlation and modeling approaches

What are longitudinal data?

For **repeated measures data**, the variable of interest is measured repeatedly for the same subjects under different conditions. Example: heart rate measurements for several subjects after different exercises.

Longitudinal data are a type of repeated measures data, for which the variable of interest is measured for several subjects **repeatedly over time**. Example: heart rate measurements for several subjects over 12 months.

[We will use the term "subject" for convenience, even if the unit of observation may also be an animal, crop field, country etc.]

Examples of longitudinal studies

- **Cohort studies** set up a cohort of people sharing some characteristic (e.g. born in the same year, free of a certain disease that is prospectively studied) and follow it over time. Often used in medicine/epidemiology, but also in other areas.

- **Panel studies** are similar to cohort studies, often collecting repeated measurements at specified time intervals, but the term is more common in the social and economic sciences. In some uses of the term, the panel is drawn to represent a cross-section of the population being studied and this sometimes involves replacement of panel members leaving the study.

- In **randomized** (clinical) **trials**, subjects are randomly assigned to treatment groups and in some trials followed up over time.

Notation and special cases

- Let n_i be the number of observations per subject for subjects $i = 1, \ldots, N$.
- Let $t_{i1}, \ldots, t_{in_i}$ be the time points where subject i is measured.
- **Balanced data** has the same number of observations $n_1 = \cdots = n_N$ and the same time points $t_{ij} \equiv t_j, j = 1, \ldots, n_i$, for all subjects i.
- If the observation times also have the same distance $d = t_{j+1} - t_j$ for all j, they are called **equally spaced**.

Some observations on longitudinal data

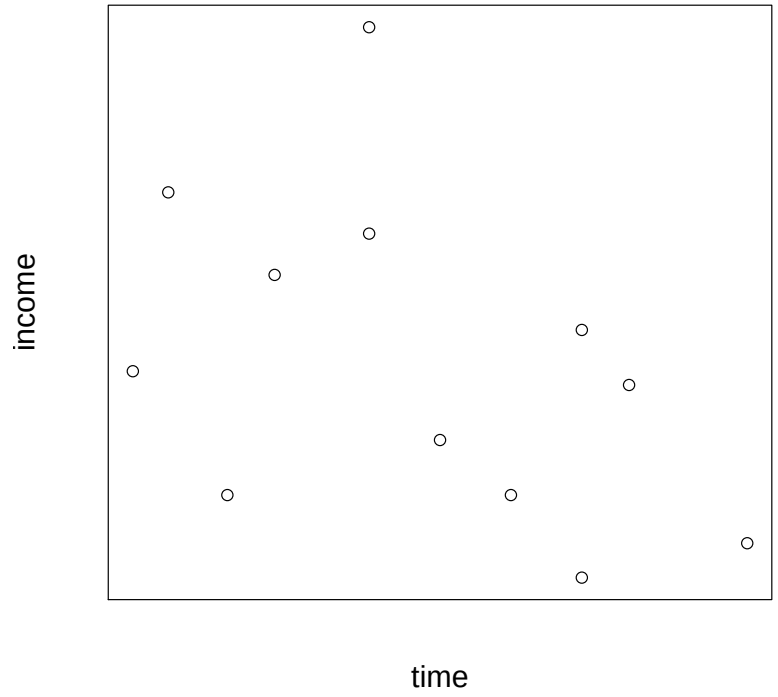
- Covariates can be **time-invariant** and only measured at baseline, e.g. gender. Or they can be **time-varying** and measured over time, e.g. physical activity.

- Sometimes longitudinal data is measured together with **survival / time-to-event data** (more in Chapter 12).

- Longitudinal data can be measured **prospectively** or **retrospectively** (e.g. via a survey or by searching through archives). Prospective studies are typically more reliable (e.g. recall bias, when for example patients who developed a disease better remember risk factors they deem important).

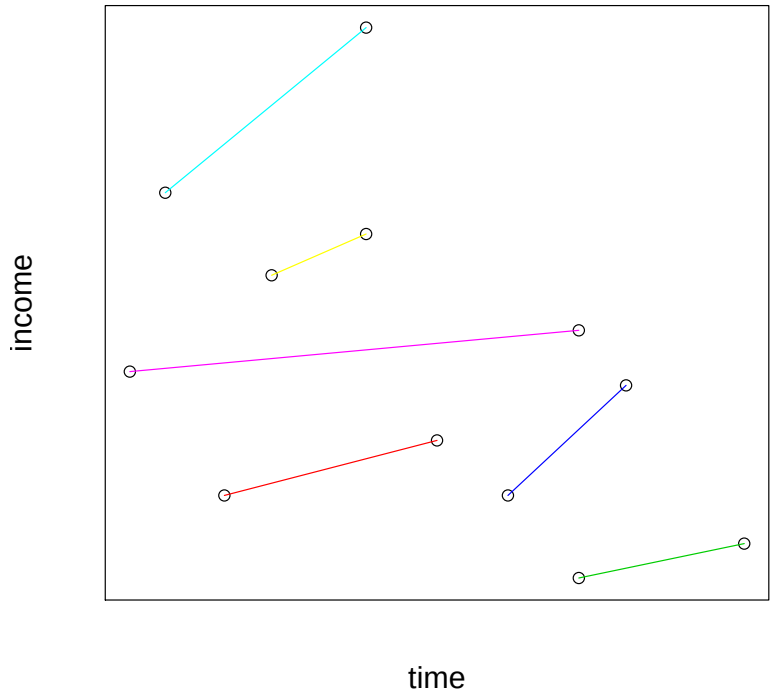
Advantages of longitudinal studies: Longitudinal effects

We can distinguish longitudinal from cross-sectional effects.



Is income decreasing over time?

Advantages of longitudinal studies: Longitudinal effects



Income is increasing over time for each person.

Starting salaries seem to be decreasing over time.

Advantages of longitudinal studies: Longitudinal effects

Longitudinal studies can follow individual change over time and are thus more informative than cross-sectional studies.

We can distinguish **cross-sectional** and **longitudinal effects**, e.g.

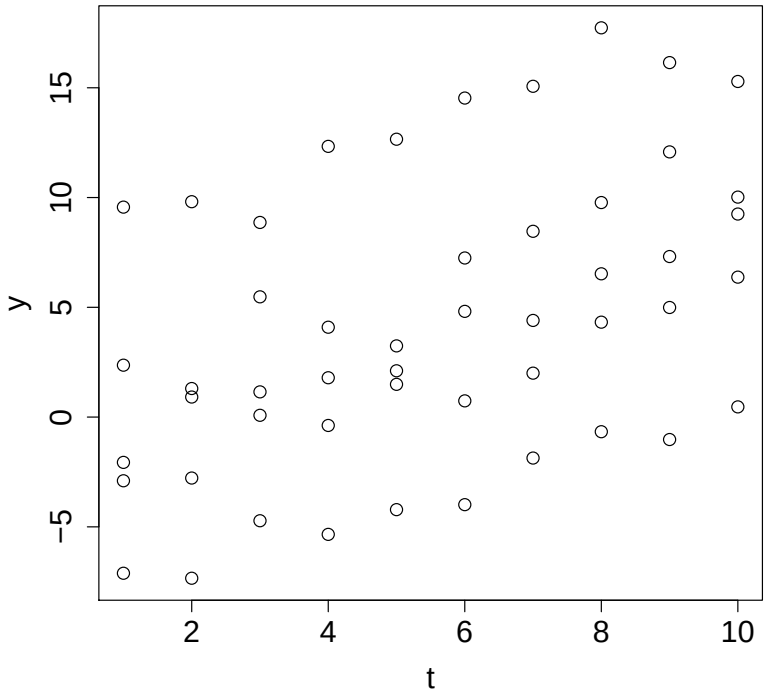
$$\mathsf{E}[Y_{ij}] = \beta_0 + \beta_C t_{i1} + \beta_L (t_{ij} - t_{i1}),$$

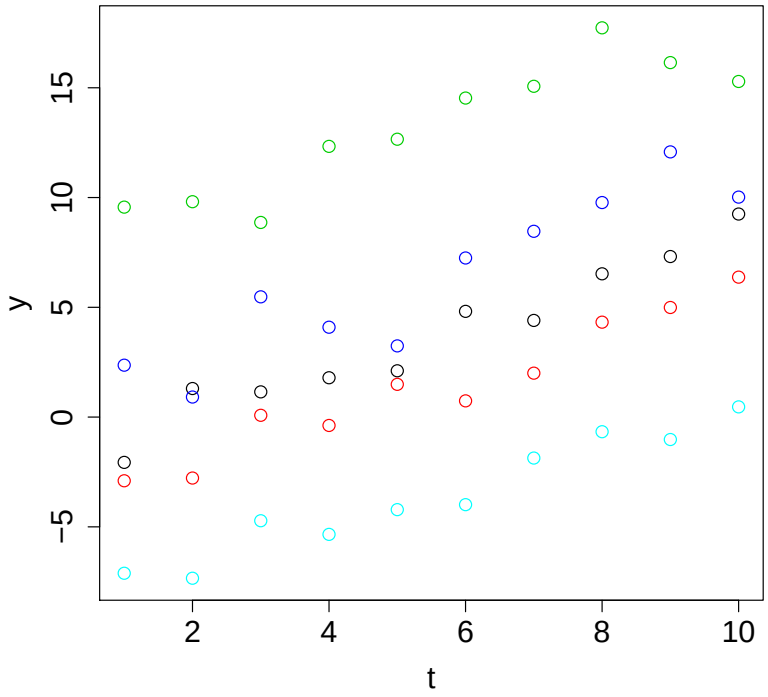
- $\beta_C =$ Increase in average starting salaries per year
- $\beta_L =$ increase in salary per year after starting to work

Without longitudinal information, we have to assume $\beta_C = \beta_L$. This is a strong assumption! In our example, β_C and β_L have opposite signs. Another example: age vs. cohort effects.

Advantages of longitudinal studies: Power

Even if $\beta_C = \beta_L$, longitudinal studies are typically more powerful than cross-sectional studies to estimate β_L.





Advantages of longitudinal studies: Confounding

Better protection against **confounding**: For changes in the response, each subject serves as its own control for time-constant variables such as age, gender, socio-economic background, education, genetics, disease history, ...

[**Confounder**: a variable that is associated with both the response and the covariate of interest and will lead to biased effect estimates if ignored.]

But even then, confounding is possible by time-varying variables.

Confounding: Example air pollution

Consider a study comparing cities with respect to their PM_{10} levels and mortality counts.

- Cross-sectional study comparing average PM_{10} levels and mortality:
  - Confounding by time-constant variables, e.g. different poverty levels, climates, . . .

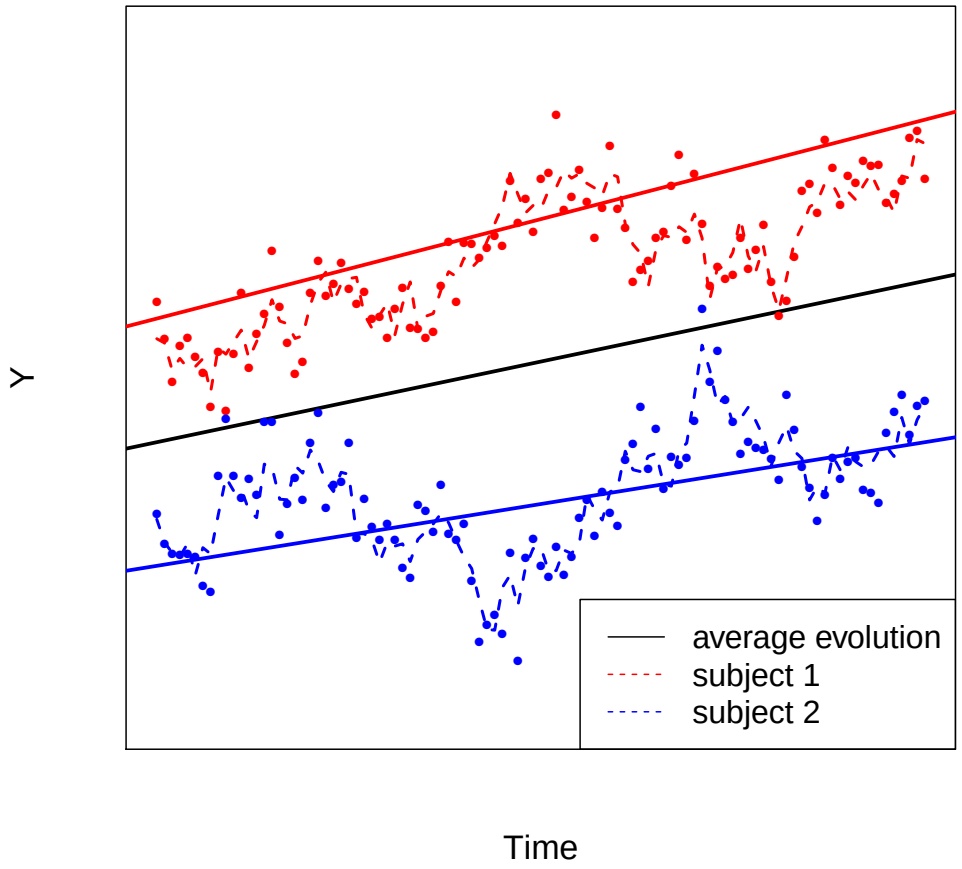
- Longitudinal study comparing daily PM_{10} levels and mortality counts:
  - No confounding by time-constant variables if each city is allowed their own average mortality level in the model.
  - Confounding by time-varying variables possible, e.g. seasonality and long-term trends.

Advantages of longitudinal studies: Sources of variation

We can distinguish different **sources of variation**:

- between subjects (**inter-subject variability**)
- within a subject over time (**intra-subject variability**).

Different sources of variability



Sources of variation: Example bloodmarker

- **Differences between people**: In average level and in average evolution over time. $\longrightarrow b_{0i}, b_{1i}$
- **Within a person over time**: Serial correlation due to e.g. long half-life of blood-marker, longer-term influences (alcohol, . . .) etc. $\longrightarrow \epsilon_{ij}^{(1)}$
- **Measurement Error**: Bloodmarker not exactly measured. $\longrightarrow \epsilon_{ij}^{(2)}$

Possible model, $\epsilon_{ij}^{(1)}$ auto-correlated, $\epsilon_{ij}^{(2)}$ i.i.d. error:

$$Y_{ij} = (\beta_0 + b_{0i}) + (\beta_1 + b_{1i})t_{ij} + \epsilon_{ij}^{(1)} + \epsilon_{ij}^{(2)}.$$

What is special about longitudinal data?

- Observations on the same subject are more similar than observations on different subjects. They are not independent, but **correlated**.
- Observations have an ordering in time.
- Often, observations are more similar the closer they are in time, i.e. the correlation is **decreasing with the time difference**. (In contrast to clustered data, e.g. on families.)
- Missing data are common, e.g. because of **drop-out**.

Some challenges in longitudinal data

- Appropriate modeling of correlation structure.
- There has been a lot of development in recent years, but flexibility and robustness of software can still be an issue.
- Missing values are methodologically challenging and constistute a problem depending on the missing data mechanism and the method used (more in Chapter 11).

Challenges in longitudinal data: Time-varying covariates

- determining an appropriate **lag structure** of covariate effects. Examples:
  - does air pollution increase mortality immediately? After hours? Days? Cumulatively?
  - carry-over effects in cross-over trials
- **covariate endogeneity** when the response predicts the covariate values at later times (**feedback** mechanisms). Examples:
  - the treatment is changed when the response values indicate that the patient is not responding
  - patients in a study on the effects of physical activity on blood glucose levels increase their physical activity after high glucose measurements.

More in Chapter 12.

Longitudinal and other data

- (Balanced) longitudinal data can be viewed as a type of **multivariate data**. But with a special correlation structure!
- **Hierarchical / multi-level / clustered data**: Similar nested structure and approaches (random effects etc.), but without the temporal structure.
- **Spatial data**: 2-D / 3-D, no inherent ordering, usually no independent subunits. But many similar approaches to modeling correlation: Marginal models, Gaussian random effects / fields, Markov chains / random fields
- Longitudinal data can be viewed as realizations of **stochastic processes**.

Analysis of longitudinal data (ALD) vs. time series analysis

- Both model time courses and try to take into account temporal **correlation** between observations.
- In contrast to time series analysis, ALD usually focuses on the estimation of **covariate effects**.
- Longitudinal data typically span shorter time periods than time series, but they contain **independent replications** in the form of subjects. This allows us to borrow strength (can be more robust to model assumptions).
- Many concepts from time series analysis are useful in ALD.

Typical questions with longitudinal data

- Are there changes over time?
- If so, which shape do they take? Linear? Are there break points?
- Do changes depend on covariates, e.g. on treatment group or gender?
- Are changes associated with the baseline value at $t = 0$?
- How large is the intra-individual variability compared to the inter-individual variability?

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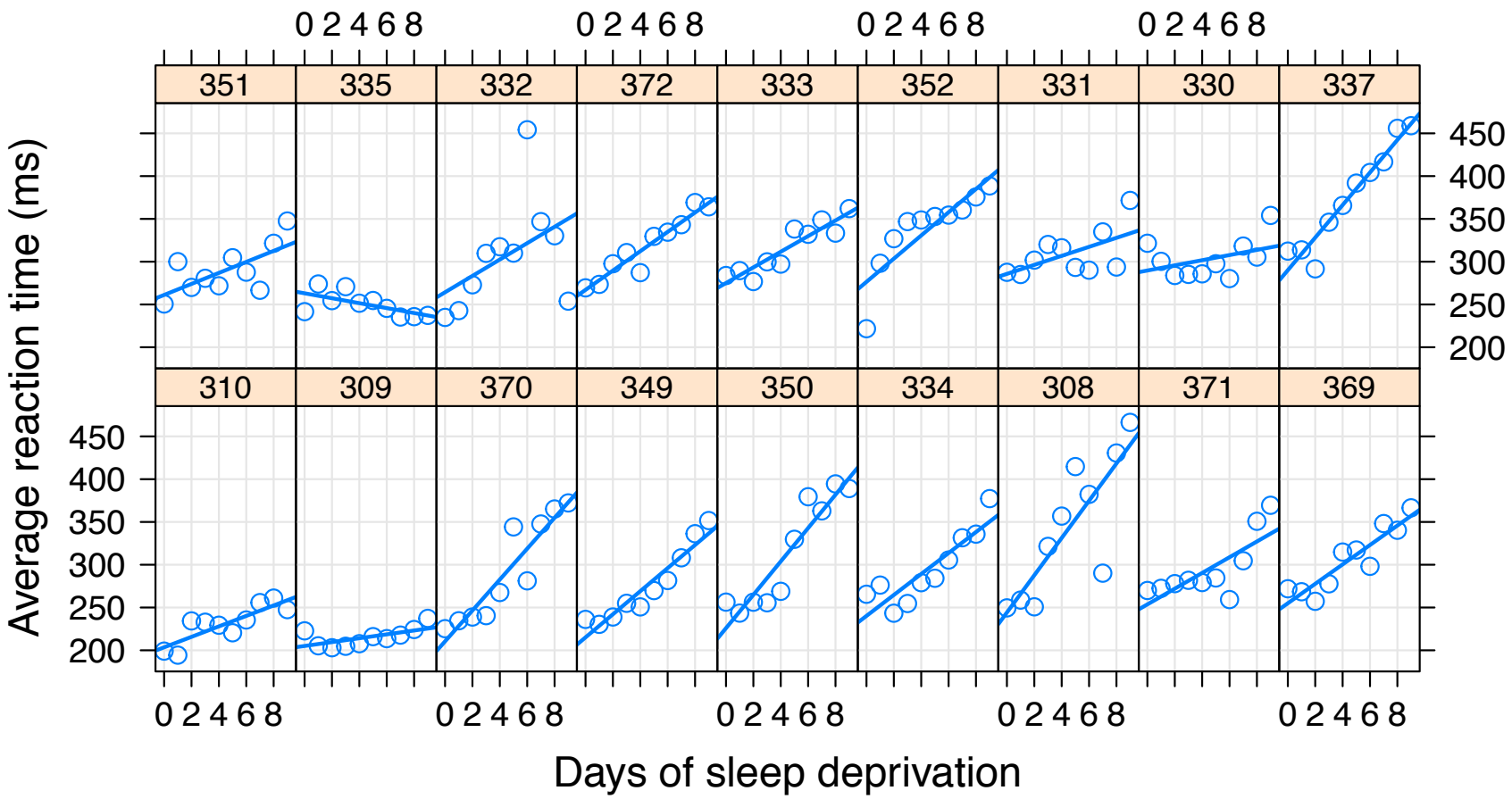
Example 1: Sleep deprivation study

- Sleep deprivation study with daily measurements from day 0 (normal sleep) to day 8 (3 hours sleep per night on subsequent nights) for $N = 18$ subjects.
- Response: average reaction time (in milliseconds, ms) on a series of tests
- No missings, balanced and equally spaced data
- First analyzed in Belenkey et al (2003), re-analyzed in Bates et al (2014) and part of the R-package `lme4`.

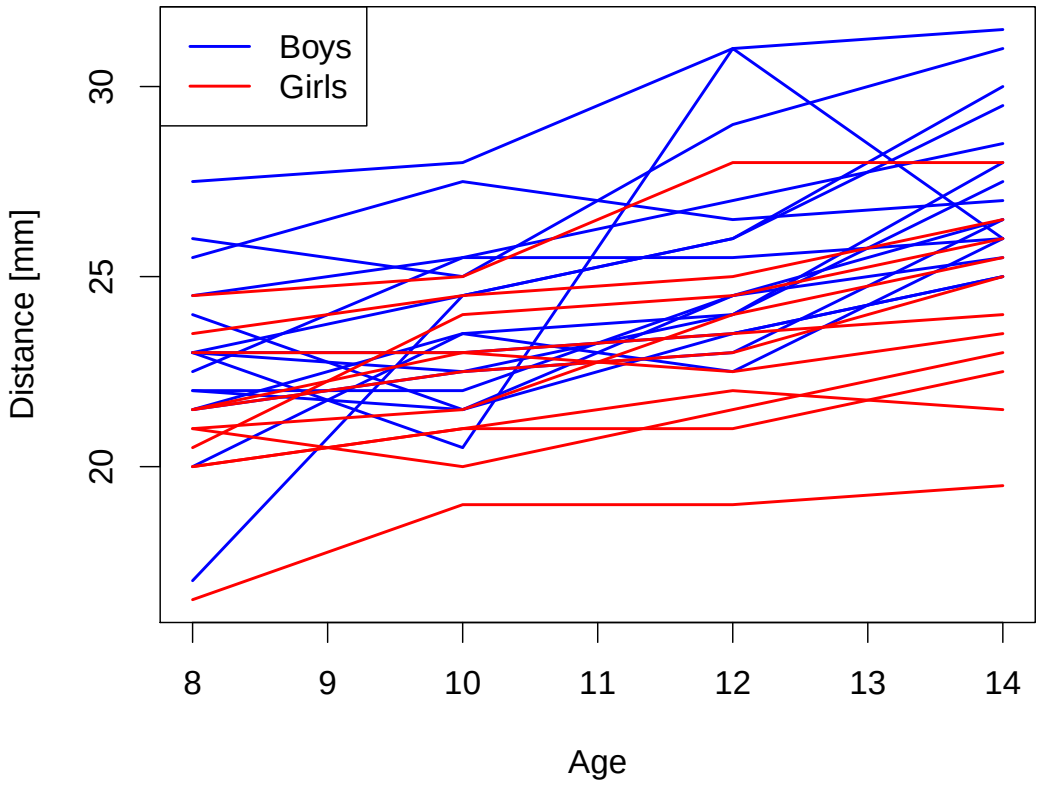
Example 1: Sleep deprivation study



Example 2: Growth in children (Orthodont data)

- Data from Potthoff and Roy (1964), re-analyzed in the book by Little and Rubin (1987) and part of the R-package `nlme`.
- 11 girls, 16 boys
- Response: distance between two points in the face (in mm)
- 4 measurements at the ages 8, 10, 12, 14 (balanced data, equally spaced)
- **Questions of interest:** Comparison of intercept and slope between boys and girls. Heterogeneity between subjects?

Example 2: Growth in children (Orthodont data)



Example 3: Treatment of lead-exposed children (TLC)

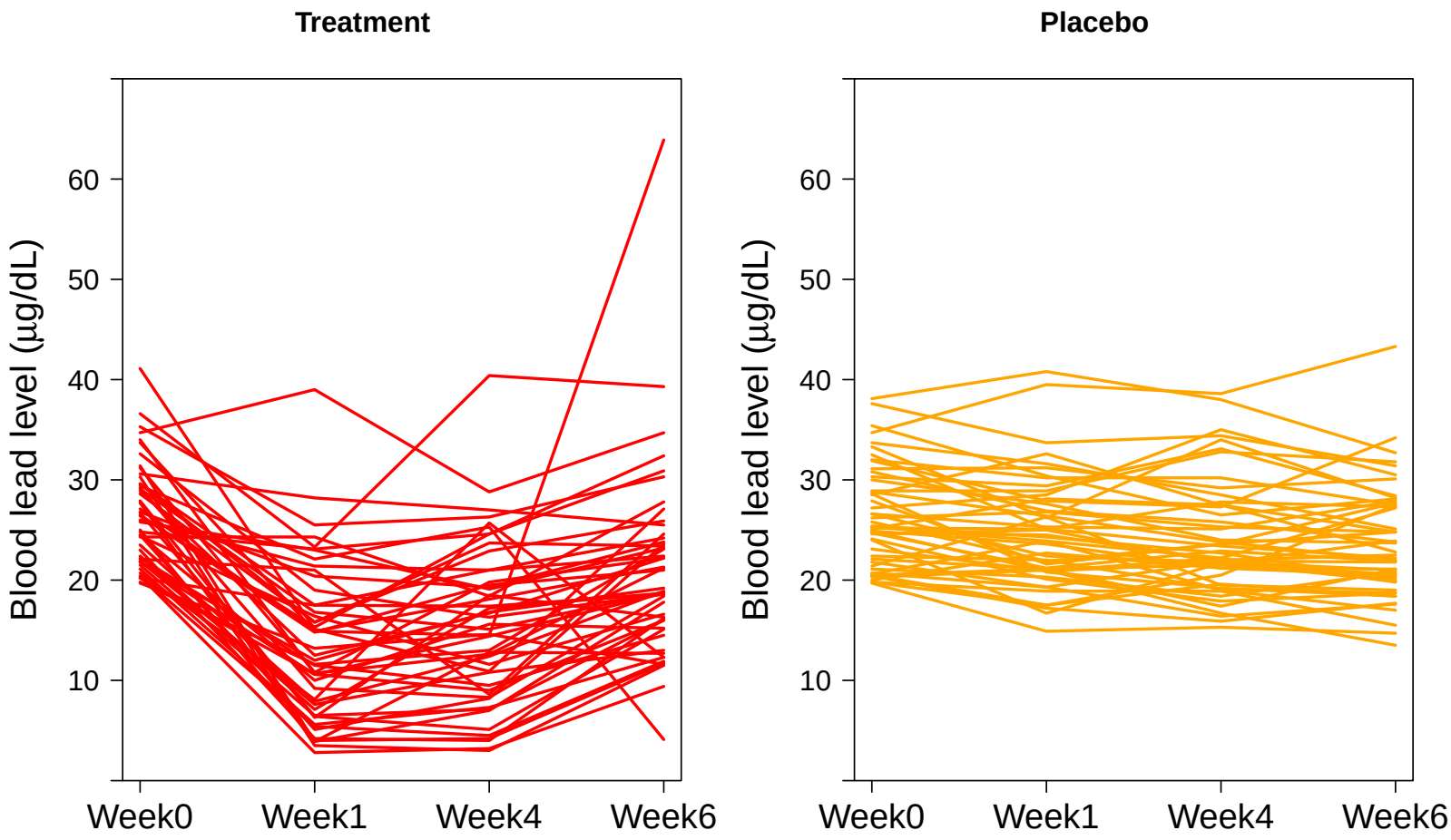
Background: US children can be exposed to lead-based paint in deteriorating housing from before 1978 (when the paint was banned). High blood levels of lead result in risk of several adverse health effects.

TLC trial (see Fitzmaurice et al, 2004): In this data set

- N=100 children 12-33 months old with high blood lead levels
- Response: Blood lead level (μg/dL)
- Treatment: placebo or succimer (enhances urinary excretion of lead)
- Measurements: baseline, week 1, week 4 and week 6 (balanced data).

Data source: `http://www.hsph.harvard.edu/fitzmaur/ala/tlc.txt`

Example 3: TLC trial

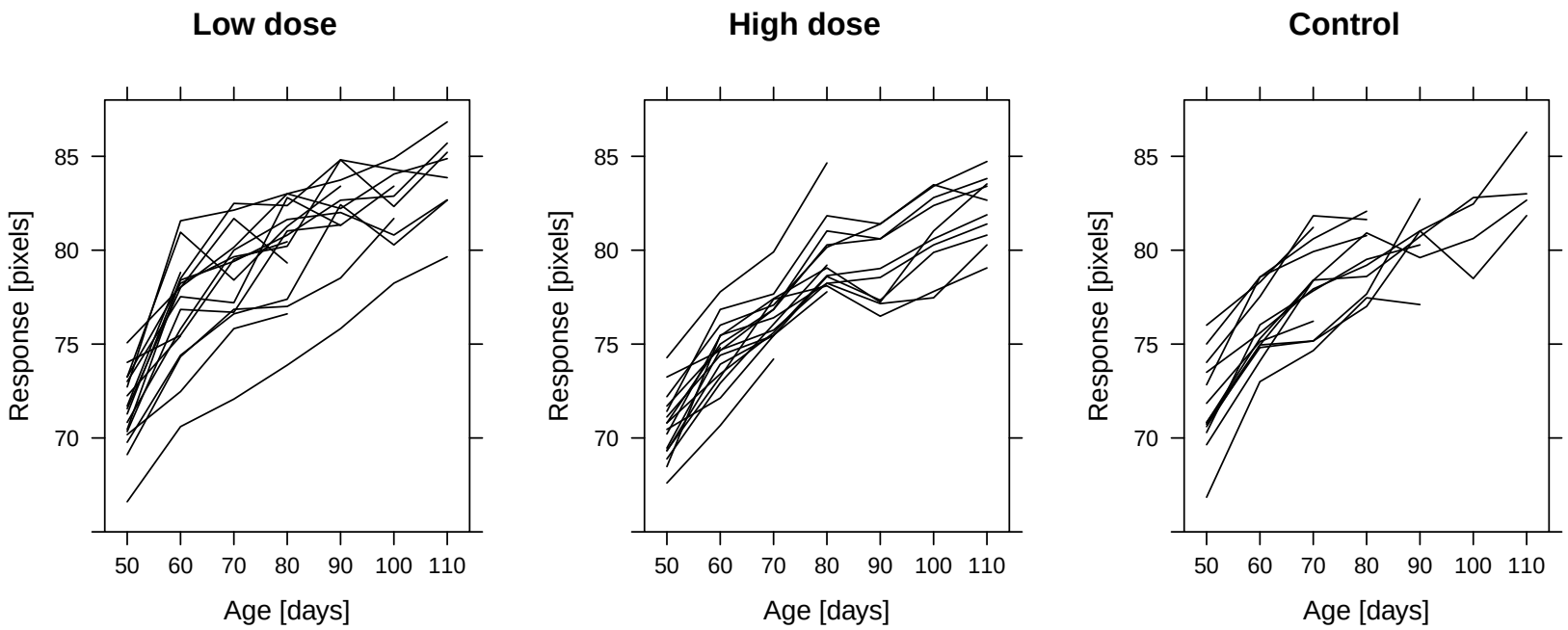


Example 4: Rats

Question: Effect of an inhibitor for testosterone production in rats on their craniofacial growth (see Verbeke and Molenberghs, 2000, `https://perswww.kuleuven.be/~u0018341/documents/rats.sas`).

- $N = 50$ male rats
- randomized into three groups: control, low dose, high dose
- **Response:** Distance between two well-defined points on X-ray pictures of the skull, characterizing the height of the skull (in pixels)
- Same measurement times t_j for all rats, but **dropout** due to rats not surviving the anesthesia (unequal n_i).

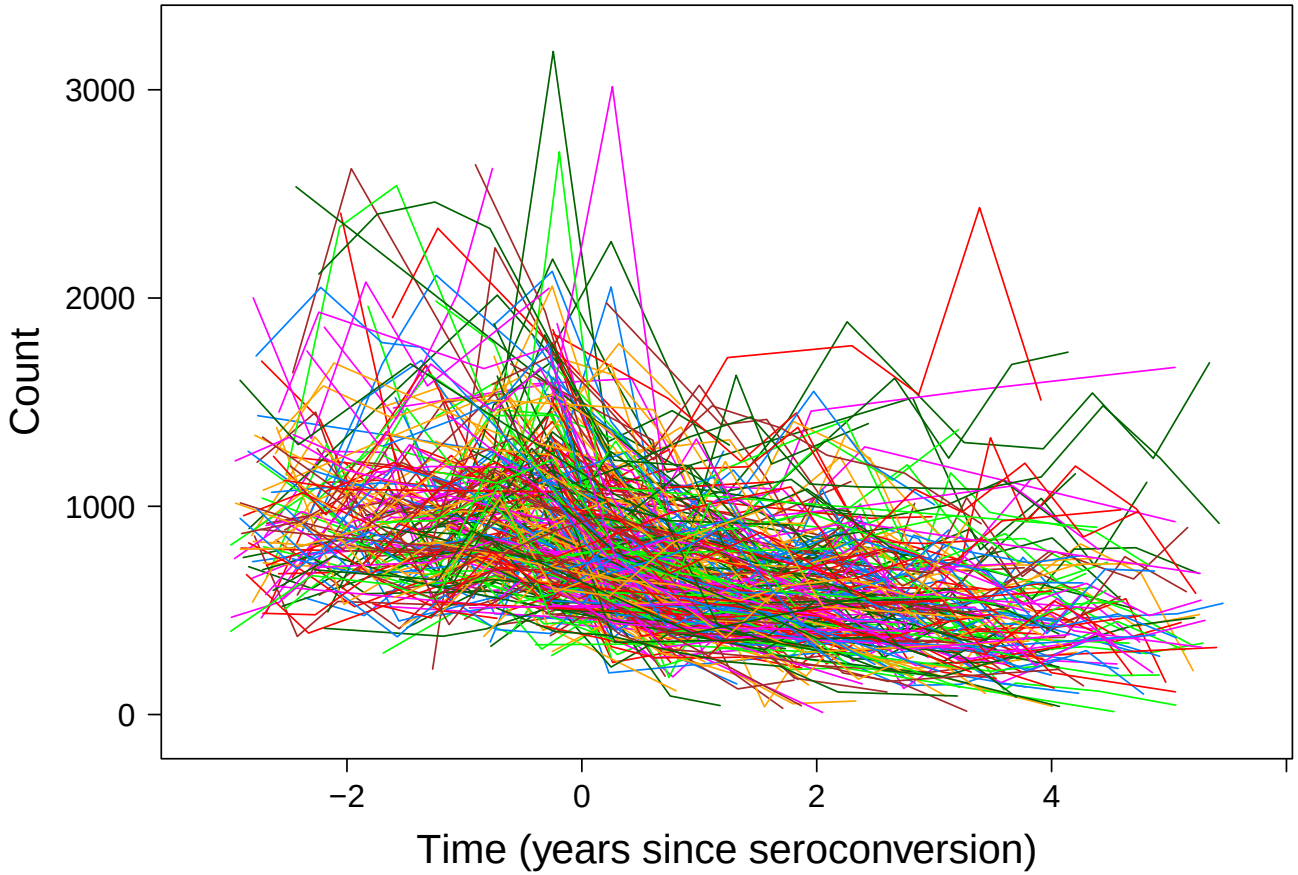
Example 4: Rats



Example 5: CD4

- **Background:** The human immunodeficiency virus (HIV) destroys CD4 cells, which are important in the body's immunoresponse. The CD4 cell count decreases after seroconversion and is a good indicator for the disease development.
- The data with 2376 observations on 369 men infected with HIV is highly unbalanced (see Diggle et al, 2002).
- **Questions of interest:**
  - the average time course for the CD4 cell depletion
  - time courses for individual men
  - heterogeneity between men
  - factors influencing the CD4 cell count change (info on age, cigarette and drug use, number of sexual partners, psychological health)

Example 5: CD4



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Why are simple methods not adequate?

Example orthodont data. Question: Difference between genders in change over time? Possible naive approaches:

- Linear regression model with covariates gender, age and their interaction.
- Cross-sectional analysis, comparison of boys and girls at each age
- Linear regression model with covariate age for each subject. Comparison of subject-specific regression coefficients between boys and girls.

Different viewpoints of correlation

- **Marginal models**: Model marginal correlation and/or account for it with robust standard errors (GEE).
- **Mixed models**: Observations are correlated, because they are from the same subject and share the same underlying processes.
- **Transition/Markov models**: Observations are correlated, because the past influences the presence.
  (Typical here: Past $=$ last q observations $\longrightarrow$ Markov property.)

The three approaches for the linear model

Consider a simple linear regression model (e.g. for child growth, $Y =$ height)

$$\mathsf{E}[Y_{ij}] = \beta_0 + \beta_1 t_{ij}. \tag{1}$$

- **Marginal model**: In addition to (1), specify a model for variance $\mathsf{Var}(Y_{ij})$ and correlation $\mathsf{Corr}(Y_{ij}, Y_{ik})$.

- **Mixed model**: Models curves with subject-specific means, e.g.

$$Y_{ij} = (\beta_0^* + b_{i0}) + (\beta_1^* + b_{i1}) t_{ij} + \epsilon_{ij}$$

$$\begin{pmatrix} b_{i0} \\ b_{1i} \end{pmatrix} \overset{iid}{\sim} \mathcal{N}\left(\mathbf{0}, \begin{pmatrix} d_{11} & d_{12} \\ d_{12} & d_{22} \end{pmatrix}\right) \text{ ind. of } \epsilon_{ij} \overset{iid}{\sim} \mathcal{N}(0, \sigma^2)$$

- **Transition model**: Model the present in terms of the past, e.g. ($q = 1$)

$$\begin{aligned} Y_{ij} &= \beta_0^{**} + \beta_1^{**} t_{ij} + \epsilon_{ij} \\ \epsilon_{ij} &= \alpha \epsilon_{ij-1} + \xi_{ij}, \;\; \xi_{ij} \overset{iid}{\sim} \mathcal{N}(0, \tau^2), \epsilon_{i1} \sim \mathcal{N}(0, \sigma^2) \end{aligned}$$

- In the linear model: Transition and linear mixed model imply marginal models with particular correlation structures (cf. Ch. 3.5 and 6.1). The β parameters in all three approaches have the same marginal interpretation. This is no longer the case in the generalized setting, see Ch. 8 ff.

- For the linear case, we will focus on the linear mixed model (Ch. 3-7). The generalized linear mixed model is discussed in Chapter 9.

- Marginal models are discussed for the generalized case in Chapter 10.

Outlook

1. Introduction
2. Exploring and displaying longitudinal data
3. The longitudinal linear mixed model
4. Estimation in the LLMM
5. Inference in the LLMM
6. Flexible extensions of the LLMM
7. Model building and model choice
8. Non-normal longitudinal data
9. The generalized linear mixed model
10. Marginal models
11. Missing data
12. Selected topics