

On this exercise sheet we repeat and deepen model diagnostic and model choice in linear mixed models (lecture slides 7) and start considering non-normal longitudinal data and their modeling capabilities (lecture slides 8).

Exercise 1: Model diagnostic

In this exercise, we focus on the diagnosis of estimated linear mixed models to verify the underlying assumptions and specifications.

- (a) Download the data set `vitamin` from the homepage and read through the description. Take a first look at the data.
- (b) Estimate a linear mixed model (`m_RIRS`) with random intercepts and random slopes for each child as well as fixed effects for `time` and for the interaction of `group` and `time`. Assume that there is no serial correlation.
Note: In order to estimate no main effect for `group`, use `group*time - group` in the formula.
- (c) Why do we not need to consider the main effect for `group` here?
- (d) What will you get if you call `predict(m_RIRS)` and what does `predict(m_RIRS, level=0)` give you?
- (e) You now want to evaluate the model fit. Therefore, it is common to plot the residuals against the covariates. Which two model weaknesses can in principle be found by this?
- (f) Now look at the population-specific residuals $r_{ij} = y_{ij} - x_{ij}^\top \hat{\beta}$ and plot the residuals against the covariate `time`. Interpret the plot.
- (g) Which other plot could you look at to check for misspecifications of the mean?
- (h) Why is the consideration of a quantile-quantile plot for the residuals r_{ij} inappropriate?
- (i) Which alternatives to the residuals r_{ij} could be considered?

Exercise 2: Model choice

In this exercise, we will compare linear mixed models and select the one which is more appropriate.

- (a) In exercise 1, we have seen that the mean was not well specified, it may therefore make sense to use the transformed variable `log(time)` instead of `time`. In the following, estimate the model `m_RIRSlog` which is identical to the model fitted in exercise 1 apart from the transformation of `time`. Use **ML** (instead of REML) for the estimation and estimate model `m_RIRS` once more **using ML** as well.

Note: Keep in mind that the random slopes have to be adjusted by using the transformation as well.

- (b) How can the models `m_RIRS` and `m_RIRSlog` be compared, i.e. how can you choose which of the two models is more appropriate? To which decision do you come regarding your model selection?
- (c) What difficulty would arise if the above models, which we want to compare, were estimated by REML?
- (d) What assumption is made when considering the marginal and the conditional AIC and what can be concluded from this for their use?
- (e) In the following, consider the model `m_RIRSlog` once more but without random slopes. What decision would you expect if you compared the model with random slopes and the model without random slopes using the marginal AIC?
- (f) Instead, use the conditional AIC (included in the R package `cAIC4`) to compare the two models. To which decision do you come?

Note: In order to use the function `cAIC4`, estimate the models using the function `lmer` included in the package `lme4` (cf. lecture slides). Consider the help `?lme4` to understand the main differences compared with the already known function `lme`.

Exercise 3: Non-normal longitudinal data

In the following, we are leaving the normal distribution assumption of \mathbf{Y}_i and allow that the distribution of the data belongs to the exponential family. This is analogous to the transition from linear models to generalized linear models (GLM).

- (a) As a first step, consider a couple of own examples of **non-normal longitudinal** data (not the ones from the lecture!).
- (b) In the data set `epi1` in the R package `MASS`, the numbers of seizures over time are available for 59 epileptics. The probands were randomly assigned to a treatment group receiving the drug Progabide and a placebo group.
- (i) Which model would you choose if you were interested in individual predictions for the probands?

- (ii) Which model would you choose if you were interested in the population effect of the drug Progabide?
- (iii) Let $\hat{\beta}$ be the estimated effect of Progabide derived from the model fitted in (i). What do you have to consider regarding the interpretation of $\hat{\beta}$? Does the interpretation correspond to the one of the effect of Progabide derived from the model in (ii)?