

Sheet 6, Exercise 1 (11):

We have the following relation:

$$\log \left(\frac{P(Y_{ij} = 1 | b_i, \mathbf{sex}_i, \mathbf{month}_{ij})}{P(Y_{ij} = 0 | b_i, \mathbf{sex}_i, \mathbf{month}_{ij})} \right) = \beta_0 + \mathbf{month}_{ij}\beta_1 + \mathbf{sex}_i\beta_2 + b_{i0} = \eta_{ij}$$

For the effect of **sex**, we consider the log-odds ratio:

$$\begin{aligned} & \log \left\{ \left(\frac{P(Y_{ij} = 1 | b_i, \mathbf{sex}_i = 1, \mathbf{month}_{ij})}{P(Y_{ij} = 0 | b_i, \mathbf{sex}_i = 1, \mathbf{month}_{ij})} \right) / \left(\frac{P(Y_{ij} = 1 | b_i, \mathbf{sex}_i = 0, \mathbf{month}_{ij})}{P(Y_{ij} = 0 | b_i, \mathbf{sex}_i = 0, \mathbf{month}_{ij})} \right) \right\} \\ &= \log \left(\frac{P(Y_{ij} = 1 | b_i, \mathbf{sex}_i = 1, \mathbf{month}_{ij})}{P(Y_{ij} = 0 | b_i, \mathbf{sex}_i = 1, \mathbf{month}_{ij})} \right) - \log \left(\frac{P(Y_{ij} = 1 | b_i, \mathbf{sex}_i = 0, \mathbf{month}_{ij})}{P(Y_{ij} = 0 | b_i, \mathbf{sex}_i = 0, \mathbf{month}_{ij})} \right) \\ &= \beta_0 + \mathbf{month}_{ij}\beta_1 + \beta_2 + b_{i0} - \beta_0 - \mathbf{month}_{ij}\beta_1 - b_{i0} \\ &= \beta_2 + b_{i0} - b_{i0} \end{aligned}$$

Thus, this is only equal β_2 when the random effects cancel out (are equal).